Homework 11

1. Suppose a linear harmonic oscillator is driven with a period force represented by a Fourier series, so that its steady **response** may be written

$$x(t) = \sum_{n=0}^{\infty} D_n \cos(n\omega t - \delta_n).$$

a) Why is it possible to include only cosine terms in this series? (Hint: what do you get if you add a cosine and sine of the same frequency?)

b) Find the root-mean-square (rms) displacement of the oscillator, in terms of the D_n. The rms displacement is

$$\langle x^2 \rangle = \frac{1}{\tau} \int_{period} x^2 dt.$$

The result you derived is called "Parseval's Theorem".

2,3,4:

5-11. Consider a massive body of arbitrary shape and a spherical surface that is exterior to and does not contain the body. Show that the average value of the potential due to the body taken over the spherical surface is equal to the value of the potential at the center of the sphere.

5-12. In the previous problem, let the massive body be inside the spherical surface. Now show that the average value of the potential over the surface of the sphere is equal to the value of the potential that would exist on the surface of the sphere if all the mass of the body were concentrated at the center of the sphere.

5-21. A point mass m is located a distance D from the nearest end of a thin rod of mass M and length L along the axis of the rod. Find the gravitational force exerted on the point mass by the rod.

5-21 b) Suppose the point mass is one-tenth the mass of the rod and is a distance 10L away from the center of the rod. The point mass is orbiting the rod, which rotates at the orbital frequency (so that it always points toward the mass.)

What is the tension in the rod? Does it vary along the length of the rod?
(You may use the same approximations we used in finding tidal force.)

5. Treat t as the independent variable.

Given $f[q,\dot{q};t] = tq\dot{q}^2$

Find the following. (use $\dot{q}$ for dq/dt, and $\ddot{q}$ for d²q/dt².)

a) $\frac{\partial f}{\partial q}$ b) $\frac{\partial f}{\partial \dot{q}}$ c) $\frac{\partial f}{\partial t}$ d) $\frac{df}{dt}$

6.

Find the path in the x-y plane, from the point x=1, y=1 to the point x=3, y=2, that makes the integral

$\int_{1,1}^{3,2} x(1-y')^{1/2}\,dx$ stationary. You may consider paths that can be written y = y(x).

7. Find the path in the x-y plane that makes

$\int_{x1}^{x2} x(1-y'^2)\,dx$ stationary.

8. Show that the geodesic on the surface of a right circular cylinder is a segment of a helix.

9. 6-7. Consider light passing from one medium with index of refraction n_1 into another medium with index of refraction n_2 (Figure 6-A). Use Fermat's principle to minimize time, and derive the law of refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

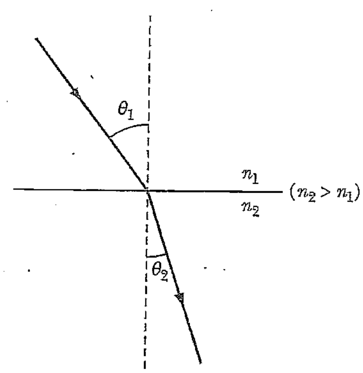

FIGURE 6-A Problem 6-7.